APPENDIX

In this section we will derive the Potential function of our game-theoretic framework for distributed spectrum allocation in cogntive radio networks.

Let $\mathbb{N} = \{1, 2, ..., N\}$ be set of Nodes and S_i be set of strategies available to node i and $i, j, k, l \in \mathbb{N}$ and $\mathrm{s}_i \in S_i, \mathrm{s}_j \in S_j$.

We define Interference function, $I_i(s_i, s_{-i})$ as:

$$I_i(s_i, s_{-i}) = \sum_{j=1, j\neq i}^{N} n_{i,j}^{-\gamma} f(s_i, s_j), \qquad \text{(i)}$$

where $\mathrm{s}_{-i} = \{\mathrm{s}_1, \mathrm{s}_2, ..., \mathrm{s}_{i-1}, \mathrm{s}_{i+1}, ..., \mathrm{s}_N\}$, n_{ij} is number of hops between node i and j

and γ is pathloss exponent,

and $f(s_i, s_j) = \begin{cases} 1 & s_i = s_j, i \neq j \\ 0 & \text{otherwise.} \end{cases}$

We define Broadcast Latency function, $T_i(s_i, s_{-i})$ as:

$$T_i(s_i, s_{-i}) = \sum_{\substack{k=1 \\ k\neq i}}^{N} \sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N} \alpha_k \beta_{i,k} \beta_{k,j} g(s_i, s_j), \qquad \text{(ii)}$$

where

$$\alpha_k = \begin{cases} 1 & \text{if } k \text{ is broadcast source} \\ 0 & \text{otherwise,} \end{cases}$$

$$\beta_{i,k} = \begin{cases} 1 & \text{if } i \text{ and } k \text{ are one-hop neighbours and } i \neq k \\ 0 & \text{otherwise,} \end{cases}$$

$g(s_i, s_j) = 1 - f(s_i, s_j)$.

The utility function is defined as:

$$U_i(s_i, s_{-i}) = -w_{a,i} I_i(s_i, s_{-i}) - w_{b,i} T_i(s_i, s_{-i}). \qquad \text{(iii)}$$

Substitute $I_i(s_i, s_{-i})$ and $T_i(s_i, s_{-i})$ from (i) and (ii) in (iii),

$$U_i(s_i, s_{-i}) = -w_{a,i} \sum_{\substack{j=1 \\ j\neq i}}^{N} n_{i,j}^{-\gamma} f(s_i, s_j) - w_{b,i} \sum_{\substack{k=1 \\ k\neq i}}^{N} \sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N} \alpha_k \beta_{i,k} \beta_{k,j} g(s_i, s_j). \qquad \text{(iv)}$$

We define Potential Function as:

$$P(s_i, s_{-i}) = -\sum_{i=1}^{N} \left(w_{a,i} \sum_{\substack{j=1 \\ j\neq i}}^{N} \lambda_a n_{i,j}^{-\gamma} f(s_i, s_j) + w_{b,i} \sum_{\substack{k=1 \\ k\neq i}}^{N} \sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N} \lambda_b \alpha_k \beta_{i,k} \beta_{k,j} g(s_i, s_j) \right).$$

$$P(s_i, s_{-i}) = -\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} w_{a,i}\lambda_a n_{i,j}^{-\gamma} f(s_i, s_j) - \sum_{i=1}^{N}\sum_{\substack{k=1\\ k\neq i}}^{N}\sum_{\substack{j=1\\ j\neq i\\ j\neq k}}^{N} w_{b,i}\lambda_b \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j). \qquad \text{(v)}$$

Let $P(s_i, s_{-i}) = -M_1 - M_2,$ (vi)

where

$$M_1 = \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} w_{a,i}\lambda_a n_{i,j}^{-\gamma} f(s_i, s_j), M_2 = \sum_{i=1}^{N}\sum_{\substack{k=1\\ k\neq i}}^{N}\sum_{\substack{j=1\\ j\neq i\\ j\neq k}}^{N} w_{b,i}\lambda_b \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j).$$

We simplify M_1 as:

$$M_1 = \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} w_{a,i}\lambda_a n_{i,j}^{-\gamma} f(s_i, s_j).$$

$$M_1 = \sum_{\substack{j=1\\ j\neq i}}^{N} w_{a,i}\lambda_a n_{i,j}^{-\gamma} f(s_i, s_j) + \sum_{\substack{l=1\\ l\neq i}}^{N}\sum_{\substack{j=1\\ j\neq l}}^{N} w_{a,l}\lambda_a n_{l,j}^{-\gamma} f(s_l, s_j).$$

$$M_1 = \sum_{\substack{j=1\\ j\neq i}}^{N} w_{a,i}\lambda_a n_{i,j}^{-\gamma} f(s_i, s_j) + \sum_{\substack{l=1\\ l\neq i}}^{N}\left(\sum_{\substack{j=1\\ j\neq l\\ j\neq i}}^{N} w_{a,l}\lambda_a n_{l,j}^{-\gamma} f(s_l, s_j) + w_{a,l}\lambda_a n_{l,i}^{-\gamma} f(s_l, s_i)\right).$$

$$M_1 = \sum_{\substack{j=1\\ j\neq i}}^{N} w_{a,i}\lambda_a n_{i,j}^{-\gamma} f(s_i, s_j) + \sum_{\substack{l=1\\ l\neq i}}^{N}\sum_{\substack{j=1\\ j\neq l\\ j\neq i}}^{N} w_{a,l}\lambda_a n_{l,j}^{-\gamma} f(s_l, s_j) + \sum_{\substack{l=1\\ l\neq i}}^{N} w_{a,l}\lambda_a n_{l,i}^{-\gamma} f(s_l, s_i).$$

Let $Q(s_{-i}) = \sum_{\substack{l=1\\ l\neq i}}^{N}\sum_{\substack{j=1\\ j\neq l\\ j\neq i}}^{N} w_{a,l}\lambda_a n_{l,j}^{-\gamma} f(s_l, s_j),$

$$\Rightarrow M_1 = \sum_{\substack{j=1\\ j\neq i}}^{N} w_{a,i}\lambda_a n_{i,j}^{-\gamma} f(s_i, s_j) + \sum_{\substack{l=1\\ l\neq i}}^{N} w_{a,l}\lambda_a n_{l,i}^{-\gamma} f(s_l, s_i) + Q(s_{-i}).$$

replacing l by j,

$$M_1 = \sum_{\substack{j=1\\ j\neq i}}^{N} w_{a,i}\lambda_a n_{i,j}^{-\gamma} f(s_i, s_j) + \sum_{\substack{j=1\\ j\neq i}}^{N} w_{a,j}\lambda_a n_{j,i}^{-\gamma} f(s_j, s_i) + Q(s_{-i}).$$

Since $f(s_j, s_i) = f(s_i, s_j), n_{j,i} = n_{i,j},$

$$\therefore M_1 = \sum_{\substack{j=1\\ j\neq i}}^{N} w_{a,i}\lambda_a n_{i,j}^{-\gamma} f(s_i, s_j) + \sum_{\substack{j=1\\ j\neq i}}^{N} w_{a,j}\lambda_a n_{i,j}^{-\gamma} f(s_i, s_j) + Q(s_{-i}).$$

$$M_1 = \sum_{\substack{j=1\\ j\neq i}}^{N}\left(w_{a,i}\lambda_a n_{i,j}^{-\gamma} f(s_i, s_j) + w_{a,j}\lambda_a n_{i,j}^{-\gamma} f(s_i, s_j)\right) + Q(s_{-i}).$$

$$M_1 = \sum_{\substack{j=1 \\ j \neq i}}^{N} \lambda_a (w_{a,i} + w_{a,j}) n_{i,j}^{-\gamma} f(s_i, s_j) + Q(s_{-i}).$$

$$\text{if} \quad \lambda_a = \frac{w_{a,i}}{w_{a,i} + w_{a,j}}, \tag{vii}$$

$$\Rightarrow M_1 = \sum_{\substack{j=1 \\ j \neq i}}^{N} w_{a,i} n_{i,j}^{-\gamma} f(s_i, s_j) + Q(s_{-i}). \tag{viii}$$

Now we simplify M_2 as:

$$M_2 = \sum_{i=1}^{N} \sum_{\substack{k=1 \\ k \neq i}}^{N} \sum_{\substack{j=1 \\ j \neq i \\ j \neq k}}^{N} w_{b,i} \lambda_b \alpha_k \beta_{i,k} \beta_{k,j} g(s_i, s_j).$$

$$M_2 = \sum_{\substack{k=1 \\ k \neq i}}^{N} \sum_{\substack{j=1 \\ j \neq i \\ j \neq k}}^{N} w_{b,i} \lambda_b \alpha_k \beta_{i,k} \beta_{k,j} g(s_i, s_j) + \sum_{\substack{l=1 \\ l \neq i}}^{N} \sum_{\substack{k=1 \\ k \neq l}}^{N} \sum_{\substack{j=1 \\ j \neq l \\ j \neq k}}^{N} w_{b,l} \lambda_b \alpha_k \beta_{l,k} \beta_{k,j} g(s_l, s_j).$$

$$M_2 = \sum_{\substack{k=1 \\ k \neq i}}^{N} \sum_{\substack{j=1 \\ j \neq i \\ j \neq k}}^{N} w_{b,i} \lambda_b \alpha_k \beta_{i,k} \beta_{k,j} g(s_i, s_j)$$

$$+ \sum_{\substack{l=1 \\ l \neq i}}^{N} \sum_{\substack{k=1 \\ k \neq l}}^{N} \left(\sum_{\substack{j=1 \\ j \neq l \\ j \neq k \\ j \neq i}}^{N} \left\{ w_{b,l} \lambda_b \alpha_k \beta_{l,k} \beta_{k,j} g(s_l, s_j) \right\} + w_{b,l} \lambda_b \alpha_k \beta_{l,k} \beta_{k,\mathrm{i}} g(s_l, s_i) \right).$$

$$M_2 = \sum_{\substack{k=1 \\ k \neq i}}^{N} \sum_{\substack{j=1 \\ j \neq i \\ j \neq k}}^{N} w_{b,i} \lambda_b \alpha_k \beta_{i,k} \beta_{k,j} g(s_i, s_j)$$

$$+ \sum_{\substack{l=1 \\ l \neq i}}^{N} \sum_{\substack{k=1 \\ k \neq l}}^{N} \sum_{\substack{j=1 \\ j \neq l \\ j \neq k \\ j \neq i}}^{N} w_{b,l} \lambda_b \alpha_k \beta_{l,k} \beta_{k,j} g(s_l, s_j) + \sum_{\substack{l=1 \\ l \neq i}}^{N} \sum_{\substack{k=1 \\ k \neq l}}^{N} w_{b,l} \lambda_b \alpha_k \beta_{l,k} \beta_{k,\mathrm{i}} g(s_l, s_i).$$

$$Let \ O(s_{-i}) = \sum_{\substack{l=1 \\ l \neq i}}^{N} \sum_{\substack{k=1 \\ k \neq l}}^{N} \sum_{\substack{j=1 \\ j \neq l \\ j \neq k \\ j \neq i}}^{N} w_{b,l} \lambda_b \alpha_k \beta_{l,k} \beta_{k,j} g(s_l, s_j),$$

$$\Rightarrow M_2 = \sum_{\substack{k=1 \\ k \neq i}}^{N} \sum_{\substack{j=1 \\ j \neq i \\ j \neq k}}^{N} w_{b,i} \lambda_b \alpha_k \beta_{i,k} \beta_{k,j} g(s_i, s_j) + \sum_{\substack{l=1 \\ l \neq i}}^{N} \sum_{\substack{k=1 \\ k \neq l}}^{N} w_{b,l} \lambda_b \alpha_k \beta_{l,k} \beta_{k,\mathrm{i}} g(s_l, s_i) + O(s_{-i}).$$

Replace l by j,

$$M_2 = \sum_{\substack{k=1 \\ k \neq i}}^{N} \sum_{\substack{j=1 \\ j \neq i \\ j \neq k}}^{N} w_{b,i} \lambda_b \alpha_k \beta_{i,k} \beta_{k,j} g(s_i, s_j) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \sum_{\substack{k=1 \\ k \neq j}}^{N} w_{b,j} \lambda_b \alpha_k \beta_{j,k} \beta_{k,i} g(s_j, s_i) + O(s_{-i}).$$

$\because g(s_i, s_j) = g(s_j, s_i),\ \beta_{i,k} = \beta_{k,i}$ *and* $\beta_{k,j} = \beta_{j,k},$

$$M_2 = \sum_{\substack{k=1 \\ k\neq i}}^{N} \sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N} w_{b,i}\lambda_b \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \sum_{\substack{k=1 \\ k\neq j}}^{N} w_{b,j}\lambda_b \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j) + O(s_{-i}).$$

Also $w_{b,\mathrm{j}}\lambda_b \alpha_k \beta_{\mathrm{j},k}\beta_{k,\mathrm{i}} g(s_j, s_i) = 0 \because \beta_{\mathrm{i,k}} = 0$ for $i = k,$ Therefore such term can be ignored,

$$M_2 = \sum_{\substack{k=1 \\ k\neq i}}^{N} \sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N} w_{b,i}\lambda_b \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j) + \sum_{\substack{j=1 \\ j\neq i}}^{N} \sum_{\substack{k=1 \\ k\neq j \\ k\neq i}}^{N} w_{b,j}\lambda_b \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j) + O(s_{-i}).$$

Changing order of summation for second term:

$$M_2 = \sum_{\substack{k=1 \\ k\neq i}}^{N} \sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N} w_{b,i}\lambda_b \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j) + \sum_{\substack{k=1 \\ k\neq j \\ k\neq i}}^{N} \sum_{\substack{j=1 \\ j\neq i}}^{N} w_{b,j}\lambda_b \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j) + O(s_{-i}).$$

Also $w_{b,i}\lambda_b \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j) = 0 \because \beta_{k,j} = 0$ for $j = k,$ Therefore such term can be ignored,

$$M_2 = \sum_{\substack{k=1 \\ k\neq i \\ k\neq j}}^{N} \sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N} w_{b,i}\lambda_b \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j) + \sum_{\substack{k=1 \\ k\neq j \\ k\neq i}}^{N} \sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N} w_{b,j}\lambda_b \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j) + O(s_{-i}).$$

$$M_2 = \sum_{\substack{k=1 \\ k\neq i \\ k\neq j}}^{N} \sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N} (w_{b,i} + w_{b,j})\lambda_b \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j) + O(s_{-i}).$$

Let $\lambda_b = \dfrac{w_{b,i}}{w_{b,i} + w_{b,j}},$ (ix)

$$\Rightarrow\ M_2 = \sum_{\substack{k=1 \\ k\neq i}}^{N} \sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N} w_{b,i}\alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j) + O(s_{-i}). \qquad \text{(x)}$$

Put values of M_1 and M_2 from (viii) and (x) in (v),

$$P(s_i, s_{-i}) = -\sum_{\substack{j=1 \\ j\neq i}}^{N} w_{a,i} n_{i,j}^{-\gamma} f(s_i, s_j) - Q(s_{-i}) - \sum_{\substack{k=1, \\ k\neq i}}^{N} \sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N} w_{b,i}\alpha_k \beta_{i,k}\beta_{k,j} g(\mathrm{s}_i, s_j) - O(s_{-i}).$$

$$P(s_i, s_{-i}) = -w_{a,i}\sum_{\substack{j=1 \\ j\neq i}}^{N} n_{i,j}^{-\gamma} f(s_i, s_j) - w_{b,i}\sum_{\substack{k=1 \\ k\neq i}}^{N} \sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N} \alpha_k \beta_{i,k}\beta_{k,j} g(s_i, s_j) - O(s_{-i}) - Q(s_{-i}). \qquad \text{(xi)}$$

Using Equation (iv) and (xi):

$P(s_i, s_{-i}) = U_i(s_i, s_{-i}) - O(s_{-i}) - Q(s_{-i}),$

Hence $P(\mathrm{s}_i, s_{-i})$ satisfy the follwing condition of exact potential function,

$P(s_i^{'}, s_{-i}) - P(\mathrm{s}_i, s_{-i}) = U_i(s_i^{'}, s_{-i}) - U_i(s_i, s_{-i}).$

Therefore $P(s_i, s_{-i})$ is an exact potetial Fuction

Subsituting, Values of λ_a and λ_b from (vii) and (ix) in (v), potential is shown to be:

$$P(s_i, s_{-i}) = -\sum_{i=1}^{N}\left(\sum_{\substack{j=1 \\ j\neq i}}^{N}\frac{w_{a,i}^2}{w_{a,i}+w_{u,j}}n_{i,j}^{-\gamma}f(s_i, s_j)\right) - \sum_{i=1}^{N}\left(\sum_{\substack{k=1 \\ k\neq i}}^{N}\sum_{\substack{j=1 \\ j\neq i \\ j\neq k}}^{N}\frac{w_{b,i}^2}{w_{b,i}+w_{b,j}}\alpha_k\beta_{i,k}\beta_{k,j}g(s_i, s_j)\right).$$